

WEEKLY TEST TYJ-02 TEST 10 RAJPUR ROAD
SOLUTION Date 06-10-2019

[PHYSICS]

1. The co-ordinates of the corners of the square are (0, 0), (2, 0), (2, 2), (0, 2). Hence,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2 \times 0 + 3 \times 2 + 5 \times 2 + 8 \times 0}{2 + 3 + 5 + 8} = \frac{16}{18} = \frac{8}{9} \text{ m}$$

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2 \times 0 + 3 \times 0 + 5 \times 2 + 8 \times 2}{2 + 3 + 5 + 8} = \frac{26}{18} = \frac{13}{9} \text{ m}$$

\therefore Co-ordinates of the centre of mass = $\left(\frac{8}{9}, \frac{13}{9}\right)$.

2. Mass of the disc removed = $\frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$

Remaining mass = $M - \frac{M}{4} = \frac{3M}{4}$

Let the origin of the co-ordinate system coincide with the centre of mass of whole disc. Now, we know that;

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

X_{CM} will be zero, when

$$m_2 x_2 = -m_1 x_1$$

$\therefore x_2 = -\frac{m_1}{m_2} x_1$

Here, $m_1 = \frac{M}{4}$, $x_1 = \frac{R}{2}$

and $m_2 = \frac{3M}{4}$ (for remaining mass)

Hence, $x_2 = -\frac{M/4}{3M/4} \cdot \frac{R}{2} = -\frac{R}{6}$

i.e., $\frac{R}{6}$ from the centre (on LHS).

3. Let ρ be the density of lead.

Then, $M = \frac{4}{3} \pi R^3 \rho =$ mass of total sphere

$$m_1 = \frac{4}{3} \pi \left(\frac{R}{2}\right)^3 \rho = \text{mass of removed part} = \frac{M}{8}$$

$$m_2 = M - \frac{M}{8} = \frac{7M}{8} = \text{mass of remaining sphere}$$

Choosing the centre of big sphere as the origin,

$$X_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$0 = \frac{\frac{M}{8} \times \frac{R}{2} + \frac{7M}{8} \times x_2}{M}$$

Solving, we get; $x_2 = \frac{-R}{14}$

i. e., centre of mass of hollowed sphere would be at a distance of $R/14$ on left of O ,

i. e., shift in centre of mass = $\frac{R}{14}$.

4. Given that the system is initially at rest,

i. e., $\vec{V}_{\text{CM}} = 0$

$$\therefore \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = 0$$

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$

or $m_1 \frac{\Delta \vec{r}_1}{\Delta t} + m_2 \frac{\Delta \vec{r}_2}{\Delta t} = 0$

or $m_1 \Delta \vec{r}_1 + m_2 \Delta \vec{r}_2 = 0$

Now, here in boat-man system if the man moves towards right the boat moves towards left.

$$\therefore m_1 \Delta r_1 = m_2 \Delta r_2 \quad \dots(i)$$

($\because \Delta r_1$ is opposite to Δr_2)

If Δr_2 is the displacement of boat relative to shore, then the displacement of man relative to shore would be $(9 - \Delta r_2)$,

i. e., $\Delta r_1 = 9 - \Delta r_2 \quad \dots(ii)$

From eqn. (i) and (ii),

$$m_1 (9 - \Delta r_2) = m_2 \Delta r_2$$

or $100(9 - \Delta r_2) = 500 \Delta r_2$

$$\therefore \Delta r_2 = \frac{100 \times 9}{600} = 1.5 \text{ m}$$

i. e., Boat moves 1.5 m relative to shore in the direction opposite to the displacement of the man.

5. Given that initially the system is at rest, so

$$\vec{V}_{\text{CM}} = 0$$

Now, as in the motion of dog no external force is applied to the system, hence

$$\vec{V}_{\text{CM}} = \text{constant} = 0$$

i.e., $\frac{m \vec{v}_1 + M \vec{v}_2}{m + M} = 0$

or $m \vec{v}_1 + M \vec{v}_2 = 0$ [as $(m + M) = \text{finite}$]

or $m \frac{\Delta \vec{r}_1}{\Delta t} + M \frac{\Delta \vec{r}_2}{\Delta t} = 0$

or $m \Delta \vec{r}_1 + M \Delta \vec{r}_2 = 0$ (as $\Delta \vec{r} = \vec{d} = \text{displacement}$)

or $md_1 - Md_2 = 0$ (as \vec{d}_2 is opposite to \vec{d}_1)

or $md_1 = Md_2$... (i)

Now, when dog moves 4 m towards shore relative to boat, the boat will shift a distance d_2 relative to shore opposite to the displacement of dog; so, the displacement of dog relative to shore (towards shore) will be,

$d_1 = 4 - d_2$... (ii)

(i.e., $d_1 + d_2 = d_{\text{rel.}} = 4$).

Putting the value of d_2 from eqn. (ii) in eqn. (i),

$md_1 = M(4 - d_1)$

or $d_1 = \frac{M \times 4}{m + M} = \frac{20 \times 4}{5 + 20} = 3.2 \text{ m}$

As initially the dog was 10 m from the shore, so now he will be $(10 - 3.2) = 6.8 \text{ m}$ from the shore.

6. Refer to question 13 Two particles collide at their centre of mass.
 \therefore Distance of CM from P

$= \frac{0.1 \times 0 + 0.3 \times 1}{0.1 + 0.3} = 0.75 \text{ m}.$

7. Under mutual attraction, the centre of mass remains at rest i.e., velocity is zero.

8. The equation of motion of the centre of mass is,

$M a_{\text{CM}} = F_{\text{ext.}}$

And as there is no external force in horizontal direction, so the centre of mass of the system does not change along horizontal direction.

For vertical motion of the centre of mass,

$(a_{\text{CM}})_y = \frac{F_{\text{ext.}}}{M} = \frac{(m_1 + m_2)g - 2T}{(m_1 + m_2)}$... (i)

Further, $a_{\text{CM}} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

$= \frac{m_1 - m_2}{m_1 + m_2} a$ [$\because \vec{a}_1 = a$ and $\vec{a}_2 = -a$]

$= \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 g$ [$\because a = \frac{m_1 - m_2}{m_1 + m_2} g$]

However, the equations of motion of two blocks are

$m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$

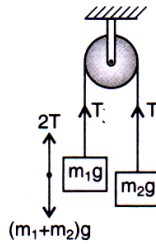
Eliminating a , we get;

$T = \frac{2m_1 m_2}{m_1 + m_2} g$... (ii)

Putting eqn. (ii) in eqn. (i), we get;

$(a_{\text{CM}})_y = \frac{(m_1 + m_2)^2 - 4m_1 m_2}{(m_1 + m_2)^2} g = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 g$

[Note: It is clear that acceleration of CM is always vertically downward irrespective of whether m_1 is heavier or m_2 .]



9. Force F_A on the particle A is given by:

$$F_A = m_A a_A = \frac{m_A v}{t} \quad \dots(i)$$

Similarly, $F_B = m_B a_B = \frac{m_B \times 2v}{t} \quad \dots(ii)$

Now, $\frac{m_A v}{t} = \frac{m_B \times 2v}{t} \quad (\because F_A = F_B)$

So, $m_A = 2m_B$

Hence, the speed of the centre of mass of the system

$$V_{CM} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{2m_B v - m_B \times 2v}{2m_B + m_B} = 0.$$

10. Here, the centre of mass of the system remains unchanged when the mass m moved a distance $L \cos \theta$, let the mass $(m + M)$ moves a distance x in the backward direction.

$$\therefore (M + m)x - mL \cos \theta = 0$$

$$\therefore x = (mL \cos \theta) / (m + M).$$

11. $R_{CM} = \frac{12 \times 0 + 16 \times 1.12 \times 10^{-10}}{12 + 16}$
 $= \frac{16}{28} \times 1.12 \times 10^{-10} \text{ m} = 0.64 \times 10^{-10} \text{ m}.$

12. $m_1 = 10 \text{ kg}, \quad m_2 = 2 \text{ kg}$

$$\vec{v}_1 = 2\hat{i} - 7\hat{j} + 3\hat{k}$$

$$\vec{v}_2 = -10\hat{i} + 35\hat{j} - 3\hat{k}$$

$$\vec{v}_{CM} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$= \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10 + 2} = 2\hat{k} \text{ m/s}$$

13. $a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

$$m_1 = m_2 = m$$

$$a_1 = 0; \quad a_2 = a$$

$$\therefore a_{CM} = \frac{ma}{2m} = \frac{a}{2}.$$

14.

15. $a = \frac{3m - m}{3m + m} g = \frac{g}{2}$

Acceleration of centre of mass

$$= \frac{3m \times \frac{g}{2} - \frac{mg}{2}}{3m + m} = \frac{g}{4}.$$

[CHEMISTRY]

16.

3Cl-atoms (E.W.G.) decrease the $-ve$ charge to a very high extent than benzene ring.

CH_3 increases electron density on \bar{C} part.

17.

Acid strength is as follows



Hence, the basicity in increasing order should be reverse for conjugated bases.



18.

Because of very high $-I$ effect NO_2 helps in the release of H^+ from $-COOH$ group, most easily.

19.

More the number of alkyl groups closer to $-O-H$ group, more is the electron density of O-atom and more is the basic nature.

20.

Aliphatic amines are more basic than aromatic amines due to increase of electron density on N-atom by their $+I$ effect.

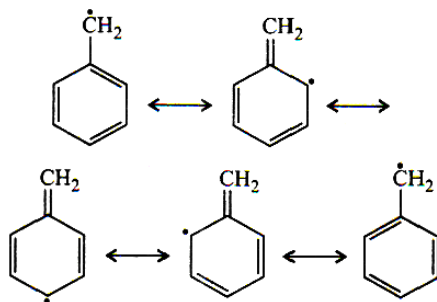
21. (b)

22.

Due to its $-I$ effect, NO_2 is decreasing the $-ve$ charge on CH_2 .

23.

$Ph\dot{C}H_2$ is the most stable among the given free radicals because of very high delocalisation of lone pair of electrons on o/p -positions of benzene ring.



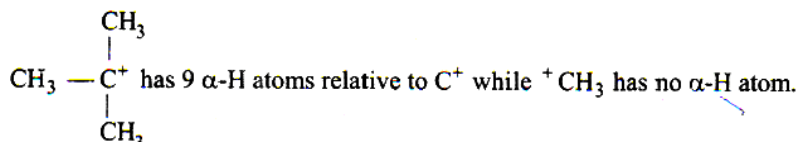
24.

$(CH_3)_3C^+$ is slightly more stable than $C_6H_5C^+H_2$.

25.

Phenyl decreases electron deficiency through resonance while CH_3 group through hyperconjugation. Resonance dominates hyperconjugation.

26.



27.

The choice (b) is non-aromatic while others are aromatic.

28.

Branches decreases boiling point.

29.

In hyperconjugation, orbital of σ -bond overlaps with p -orbital of nearby atom.

30.

Cl has $-I$ effect and increases acidic character. Moreover, closer is its position to $COOH$ group, stronger is the acid.

